eMath Solutions

Differential Equation – LSE (Management Mathematics)

Q.) Solve the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=x^2-e^x$$

Solution:

Concept used:

Steps to find the solution of a non homogeneous differential equation, which is of order two and degree one:

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$$

1.) To evaluate y_{CF} (Complementary Function)

2.) To evaluate the y_{PS} (Particular solution for particular $f(x)$)

3.) The complete solutions is

$$y=y_{CF}+y_{PS}$$

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=x^2-e^x$$

$$\Rightarrow \frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0 \quad \textit{Corresponding Homogeneous Equation}$$

$$\Rightarrow D^2+2D-3=0 \quad \textit{Corresponding Auxiliary Equation}$$

$$\Rightarrow (D+3)(D-1)=0$$

$$\Rightarrow Roots \quad D=-3,1$$

Solution to the corresponding homogeneous equation is

$$y_h=Ae^{-3x}+Be^x$$

$$For \quad f(x)=x^2-e^e$$

$$y_p=Cx^3+Dx^2+Ex+F+Ge^x$$

$$Since \quad y_h=Ae^{-3x}+Be^x \quad contains \;\; e^x$$

$$Therefore,$$

$$y=Ae^{-3x}+Bxe^x+Cx^3+Dx^2+Ex+F$$

$$To\ evaluate\ B, C, D, E\ \&\ F$$

$$differentiate\ y\ \ w.r.t.\ \ x$$

$$\frac{dy}{dx} = -3Ae^{-3x} + Bxe^{x} + Be^{x} + 3Cx^{2} + 2Dx + E$$
$$\frac{d^2y}{dx^2} = 9Ae^{-3x} + B\,x\,e^{x} + 2Be^{x} + 6C\,x + 2D$$

$$Therefore, \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = x^2 - e^x$$
$$\Rightarrow \left(9Ae^{-3x} + B\,x\,e^{x} + 2Be^{x} + 6C\,x + 2D\right) + 2\left(-3Ae^{-3x} + Bxe^{x} + Be^{x} + 3Cx^2 + 2Dx + E\right)$$
$$-3\left(Ae^{-3x} + Bxe^{x} + Cx^3 + Dx^2 + Ex + F\right) = x^2 - e^x$$
$$\Rightarrow 4Be^{x} - 3Cx^3 + (6C - 3D)x^2 + (4D - 3E + 6C)x + (2D + 2E - 3F) = x^2 - e^x$$

Giving

$$4B = -1 \quad \Rightarrow B = -\frac{1}{4}$$
$$-3C = 0 \quad \Rightarrow C = 0$$
$$6C - 3D = 1 \quad \Rightarrow D = -\frac{1}{3}$$
$$4D - 3E + 6C = 0 \quad \Rightarrow E = \left(\frac{4}{3}\right)D = -\frac{4}{9}$$
$$2D + 2E - 3F = 0 \quad \Rightarrow F = \frac{1}{3}(2D + 2E) = -\frac{2}{3}\left(\frac{1}{3} + \frac{4}{9}\right) = -\frac{14}{27}$$

Therefore complete solution is

$$y = Ae^{-3x} - \frac{1}{4}xe^{x} - \frac{1}{3}x^2 - \frac{4}{9}x - \frac{14}{27}$$