Integration

Integration

Q.) Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to(-1,1)$ be the function defined by $f'(x)=\sqrt{1-\{f(x)\}^2}$.

If $f(0)=0$, then $\int e^x f^{-1}(x)dx+\int \frac{e^x}{\sqrt{1-x^2}}dx=$ ______________

(a) $e^x\sin x+C$ (b) $e^x\sin^{-1}x+C$

(c) $e^x\cos x+C$ (d) $e^x\cos^{-1}x+C$

Solution: option (b)

Concept used:

1) Method of substitution in integration process

2) $\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x+C$

3) Integration process by parts

$$f'(x)=\sqrt{1-\{f(x)\}^2}$$
$$\Rightarrow\int\frac{f'(x)dx}{\sqrt{1-\{f(x)\}^2}}=\int dx$$
$$\Rightarrow\sin^{-1}f(x)=x+C$$
$$f(0)=0\quad\Rightarrow\quad C=0$$
$$\Rightarrow\sin^{-1}f(x)=x$$
$$\Rightarrow f(x)=\sin x$$
$$\Rightarrow f^{-1}(x)=\sin^{-1}x$$

$$\int e^x f^{-1}(x)dx+\int\frac{e^x}{\sqrt{1-x^2}}dx=\int e^x\left(\sin^{-1}x\right)dx+\int\frac{e^x}{\sqrt{1-x^2}}dx$$

Integrating by parts

$$\int e^x f^{-1}(x)dx+\int\frac{e^x}{\sqrt{1-x^2}}dx=\left[\left(\sin^{-1}x\right)e^x-\int e^x\left(\frac{1}{\sqrt{1-x^2}}\right)dx\right]+\int\frac{e^x}{\sqrt{1-x^2}}dx+C$$
$$=e^x\sin^{-1}x+C$$