

Problem on Trigonometric Equation (IB – HL)

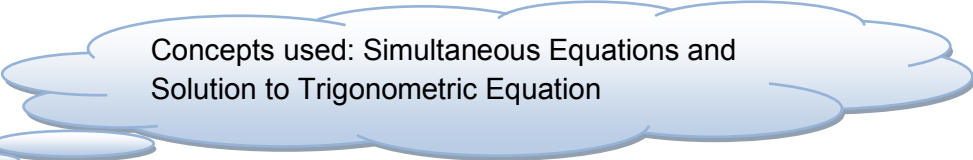
A system of equations is given by

$$\cos x + \cos y = 1.2 \quad \dots (i)$$

$$\sin x + \sin y = 1.4 \quad \dots (ii)$$

- (a) For each equation express y in terms of x .
(b) Hence solve the equation for $0 < x < \pi, 0 < y < \pi$

Solution:



Concepts used: Simultaneous Equations and
Solution to Trigonometric Equation



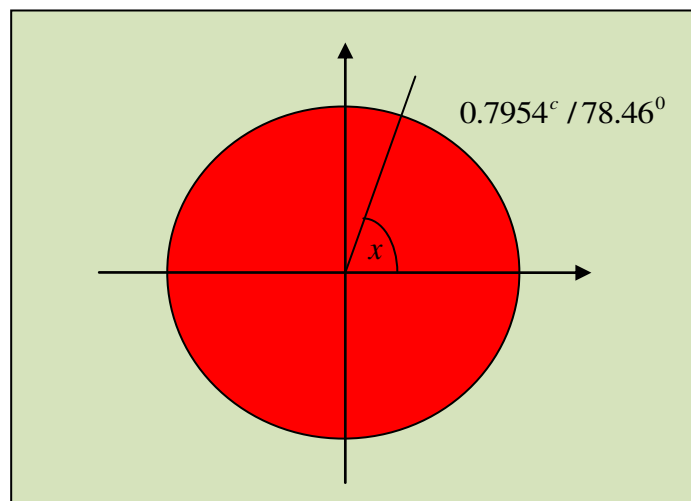
$$\cos x + \cos y = 1.2$$

$$\Rightarrow \cos y = 1.2 - \cos x$$

$$\Rightarrow y = \cos^{-1}(1.2 - \cos x) \quad \cos x \geq 0.2$$

$$\cos x \geq 0.2 \Rightarrow 0 \leq x \leq \cos^{-1} 0.2$$

$$\cos^{-1} 0.2 = 0.7954^\circ \Rightarrow 0 \leq x \leq 0.7954 \quad \dots (iii)$$



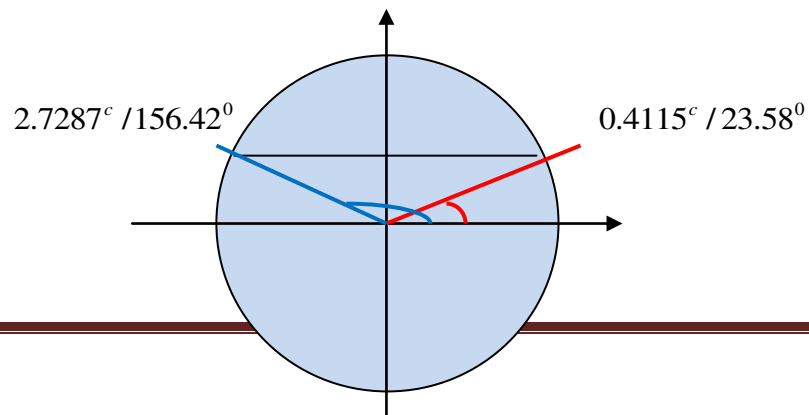
$$\sin x + \sin y = 1.4$$

$$\Rightarrow \sin y = 1.4 - \sin x$$

$$\Rightarrow y = \sin^{-1}(1.4 - \sin x) \quad \sin x \geq 0.4$$

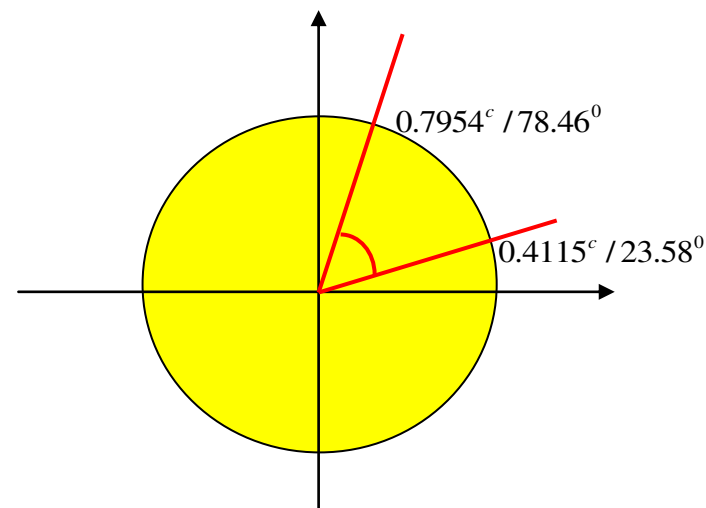
$$\sin x \geq 0.4 \Rightarrow \sin^{-1} 0.4 \leq x \leq \pi - \sin^{-1} 0.4$$

$$\sin^{-1} 0.4 = 0.4115^\circ \Rightarrow 0.4115 \leq x \leq 2.7285 \quad \dots (iv)$$



From (iii) and (iv),

$$0.4115 \leq x \leq 0.7954$$



(b) $\cos x + \cos y = 1.2 \quad \dots (i)$
 $\sin x + \sin y = 1.4 \quad \dots (ii)$

Square and add,

$$\cos^2 x + 2 \cos x \cos y + \cos^2 y = 1.44$$

$$\sin^2 x + 2 \sin x \sin y + \sin^2 y = 1.96$$

$$1 + 2(\cos x \cos y + \sin x \sin y) + 1 = 3.4$$

$$\Rightarrow 2 \cos(y - x) = 1.4$$

$$\Rightarrow \cos(y - x) = 0.7$$

$$\Rightarrow y - x = 2n\pi \pm \cos^{-1} 0.7 \quad n \in \mathbb{Z}$$

for $0 < x < \pi, 0 < y < \pi \Rightarrow n = 0$

$$\Rightarrow y - x = \cos^{-1} 0.7 = 0.7754^c / 45.57^0$$

$$\Rightarrow y = \cos^{-1} 0.7 + x = 45.57^0 + x \quad \text{where } 23.58^0 < x < 78.46^0$$